

# Compilability and Space Efficiency in Knowledge Representation

*a retrospective*  
*in ricordo di Marco Cadoli*

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# Outline of this talk

1. History and motivations
  - (a) Compilation in early '90s—from my viewpoint
  - (b) NMR in early '90s—again, from my viewpoint
2. Non-uniform Complexity Classes
3. First results: compactness vs. complexity
4. New motivations: Space efficiency
5. Compilability classes
6. Compilation goes on...—an incomplete list

# History and Motivations

# Knowledge Compilation

- In early 90's, several researchers used the word “compilation” to mean a *pre-processing* step — intended to speed-up *reasoning*
  - information used in reasoning tasks does **not** come **all at once**
- term “Knowledge Compilation” used by [Anderson, 1983] about psychological learning experiments

# Compiling $KB$ into a Horn $KB$

[Kautz and Selman, 1991a]  $\Rightarrow$  [1996]

- Propositional Knowledge Base  $KB$ 
  - deciding  $KB \models \varphi$  is  $\text{coNP}$
  - if  $KB_h$  is Horn,  $KB_h \models \varphi$  is in  $\mathcal{P}$
- idea: *compile*  $KB$  into an approximation  $KB_h$
- works if *size*  $\|KB_h\|$  is “small” wrt  $\|KB\|$
- “small”  $\approx$  poly-size

# Size of Horn compilations

- Is there an algorithm  $\mathcal{A}$  that
  - given  $KB$
  - outputs  $KB_h$  approximating  $KB$
- such that  $\|KB_h\| \leq \text{poly}(\|KB\|)$  ?
- even a nonrecursive  $\mathcal{A}$  does not exist, unless  $\Sigma_2^p = \Pi_2^p$  [Kautz and Selman, 1992]

# Compiling $KB$ into its models

[Khardon and Roth, 1994]  $\Rightarrow$  [1996]

- compile propositional  $KB$  into an efficient representation of its models
- use models to answer queries:
  - $KB \models \varphi$  iff there is no model falsifying  $\varphi$

**In the meanwhile . . .**



# Non-Monotonic Reasoning

- Circumscription [McCarthy, 1980]
- Default Logic [Reiter, 1980]
- Closed World Assumption [Minker, 1982]
- Autoepistemic Logic [Moore, 1985]
- Well-Founded Semantics for LP  
[van Gelder *et al.*, 1991]
- ...

# How many of you know... — Can I skip this slide?

## Circumscription of Knowledge Base $KB$

- model of  $KB$  = set of atoms  $M$  (true atoms)
- *minimal*  $M$  = no subset of  $M$  is a model of  $KB$
- models of  $CIRC(KB)$  = *minimal models* of  $KB$
- $Models(a \vee b) = \{\{a\}, \{b\}, \{a, b\}\}$
- $Models(CIRC(a \vee b)) = \{\{a\}, \{b\}, \{\cancel{a}, \cancel{b}\}\}$

# Why NMR?

(from Marco's PhD thesis [Cadoli, 1995])

In case of fire  
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- How one derives that there is no fire?
- $KB = \{fire \equiv (smoke \vee alarm \vee \dots)\}$
- $CIRC(KB) \models \neg fire$

# Why NMR? – 2

[McCarthy, 1980]

“Circumscription is a rule of conjecture that can be used by a person or program for ‘**jumping to certain conclusions**’ ”

[Reiter, 1980]

“... default reasoning... corresponds to... patterns of inference of the form ‘**in the absence of any information to the contrary, assume...**’ ”

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- undecidable problems  $\Rightarrow$  “more” undecidable
  - r.e.-complete  $\Rightarrow \Pi_2^p$ -complete [Schlipf, 1987]

# So, why NMR?

[Brachman, 1990]:

“An irony of work on NMR is that, while the easy adoption and retraction of assumptions is most useful for **speeding up** natural everyday reasoning, most current NMR proposals drastically compound the already difficult problem of deductive reasoning. We urgently need to determine how NMR can be used to make commonsense inference **faster, not slower.**”

# Intuition about NMR

[Cadoli *et al.*, 1994]  $\Rightarrow$  [1996b]

- “Is intractability of non-monotonic reasoning a real drawback?”
- **No**: complexity of NMR is balanced by savings in *space* when representing knowledge
- (... How to prove this claim?)

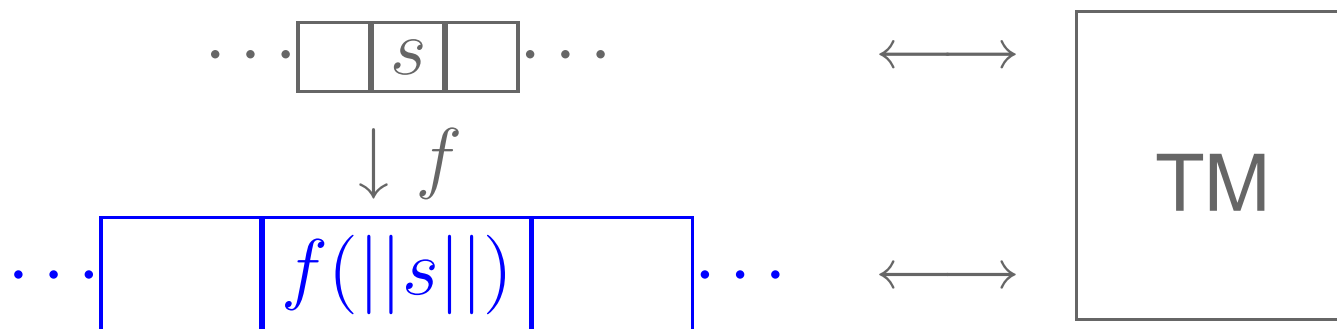
# **Techniques: Non-uniform complexity classes**



# Advice-taking Turing machines

cfr. [Johnson, 1990]

- two tapes: normal tape, **advice tape**
- function  $f$  (not necessarily recursive)
- on input  $s$ , load advice tape with  $f(\|s\|)$
- computation proceeds as normal, based on both  $s$  and  $f(\|s\|)$



# Polynomial advice

An advice-taking Turing machine uses *polynomial advice* if . . .

- condition on the advice oracle  $f$ :
- $\exists$  poly s.t.  $\|f(n)\| \leq \text{poly}(n)$  for all  $n = 1, 2, 3, \dots$
- *one* polynomial (always the same)

# Non-uniform polynomial hierarchy

- $\mathcal{C} \in \{\mathcal{P}, \mathcal{NP}, \text{co}\mathcal{NP}, \Sigma_2^p, \Pi_2^p, \dots\}$  (polynomial hierarchy)
- $\mathcal{C}/\text{poly} = \mathcal{C}$  augmented by polynomial advice
  - e.g.  $\mathcal{NP}/\text{poly} = \mathcal{NP} +$  a poly guess based on the *size* of the input
- *Non-uniform* polynomial hierarchy:
  - $\mathcal{P}/\text{poly}, \mathcal{NP}/\text{poly}, \Sigma_2^p/\text{poly}, \dots$

# Complexity results

- if  $\mathcal{NP} \subseteq \mathcal{P}/\text{poly}$  then  $\Sigma_2^p = \Pi_2^p$   
[Karp and Lipton, 1980]
- if  $\Sigma_n^p \subseteq \Pi_n^p/\text{poly}$  then  $\Sigma_{n+2}^p = \Pi_{n+2}^p$  [Yap, 1983]
- Conclusion:
  - one cannot use polynomial advice to *compile* time complexity unless ...
  - ... the polynomial hierarchy *collapses* at some level

# First Results

# Compilation

- generalization of knowledge compilation
- [Cadoli *et al.*, 1994] “We call *compilation* any off-line process that makes on-line reasoning polynomial”
- More generally, compilation should *decrease* complexity
  - e.g. from  $\Pi_2^p$  to  $\text{coNP}$
  - a *lower* class in the polynomial hierarchy

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 $CIRC(KB)$
  - pro:  $CIRC()$  saves an exp. amount of space
- $CIRC()$  *compacts* knowledge

# Compiling CIRC – 2

[Cadoli *et al.*, 1995b]  $\Rightarrow$  [1997]

- $KB$  a propositional formula
  - $KB \models \varphi$  is in  $\text{coNP}$
  - $\text{CIRC}(KB) \models \varphi$  is  $\Pi_2^p$ -complete  
[Eiter and Gottlob, 1993]
- $\text{CIRC}(KB)$  could be compiled to a *formula* iff  $\mathcal{NP} \subseteq \mathcal{NC}^1/\text{poly}$
- $\text{CIRC}(KB)$  could be compiled to a *data structure* iff  $\mathcal{NP} \subseteq \mathcal{P}/\text{poly}$

# Compiling Belief Revision

- $P$  new knowledge
- $KB \wedge P \models \perp$  (inconsistent)
- $KB * P = KB$  *revised* with  $P$
- Express  $KB * P$  as an equivalent formula  $KB'$
- does always exist  $KB'$  poly-size wrt  $KB, P$  ?

# Compiling Belief Revision

[Cadoli *et al.*, 1995a]  $\Rightarrow$  [1999]

Revision defined by...	General $P$		Bounded $P$	
	Same atoms	New Atoms	Same atoms	New Atoms
GFUV, Nebel	NO	NO	NO	NO
Winslett	NO	NO	YES	YES
Borgida	NO	NO	YES	YES
Forbus	NO	NO	YES	YES
Satoh	NO	NO	YES	YES
Dalal	NO	YES	YES	YES
Weber	NO	YES	YES	YES
WIDTIO	YES	YES	YES	YES

GFUV = Ginsberg — Fagin, Ullman, Vardi

WIDTIO = “When in doubt, throw it out”



# **New Motivations: Space Efficiency of KR formalisms**

# Representing knowledge

- **Assumption:**  $KB$  in a formalism  $F$  represents either:
  - the set of *models* of  $KB$  in  $F$ , or
  - the set of *theorems* derivable from  $KB$  in  $F$
- Compare succinctness of propositional KR formalisms

# Comparing KR formalisms

Same “piece of knowledge” can be expressed as:

- $KB_1$  in formalism  $F_1$
- $KB_2$  in formalism  $F_2$
- *Relative* space efficiency:
  - compare  $||KB_1||$  and  $||KB_2||$

# Translating knowledge

formalism $F_1$	shortest translation $f$ (even an oracle)	formalism $F_2$
$KB_1^1$	$\mapsto$	$KB_2^1$
$KB_1^2$	$\mapsto$	$KB_2^2$
$\vdots$	$\vdots$	$\vdots$

- if  $\|KB_2^i\|$  grows exponentially wrt  $\|KB_1^i\|$ ,
- and not vice versa, then
- $F_1$  is **relatively more space efficient** than  $F_2$

# Poly-size reductions among formalisms

$$f : F_1 \mapsto F_2$$

- the result of  $f(KB)$  is poly-size wrt  $\|KB\|$
- for any given  $KB$  in  $F_1$ ,  $f(KB)$  is a knowledge base in  $F_2$

# Reductions preserving models

- $f$  preserves models if

$$\forall KB : \exists g : \forall M : \left[ \begin{array}{ccc} M & \models_{F_1} & KB \\ g \downarrow & & \downarrow f \\ g(M) & \models_{F_2} & f(KB) \end{array} \right]$$

- where  $g$  is poly-time ("easy to compute")
- Note:  $g$  depends on  $KB$

# Example

[Etherington, 1987]

PFN-Skeptical Default Logic  $\mapsto$  Circumscription

$$\left[ \begin{array}{ccc} & \exists \text{extension} & \\ M & \models_{SK} & \langle \{ \frac{\neg \gamma_i}{\gamma_i} \}, W \rangle \\ g \downarrow & & \downarrow f \\ M \cup \{ a_i \mid M \models \gamma_i \} & \models_{CIRC} & W \cup \{ a_i \equiv \neg \gamma_i \} \\ & \text{minimizes } a_i & \end{array} \right.$$

# Reductions preserving theorems

- $f$  preserves theorems if

$$\forall KB : \exists g : \forall Q : \left[ \begin{array}{ccc} KB & \vdash_{F_1} & Q \\ f \downarrow & & \downarrow g \\ f(KB) & \vdash_{F_2} & g(Q) \end{array} \right]$$

- where  $g$  is poly-time and  $\|Q\| \leq \|KB\|$
- Note:  $g$  depends on  $KB$

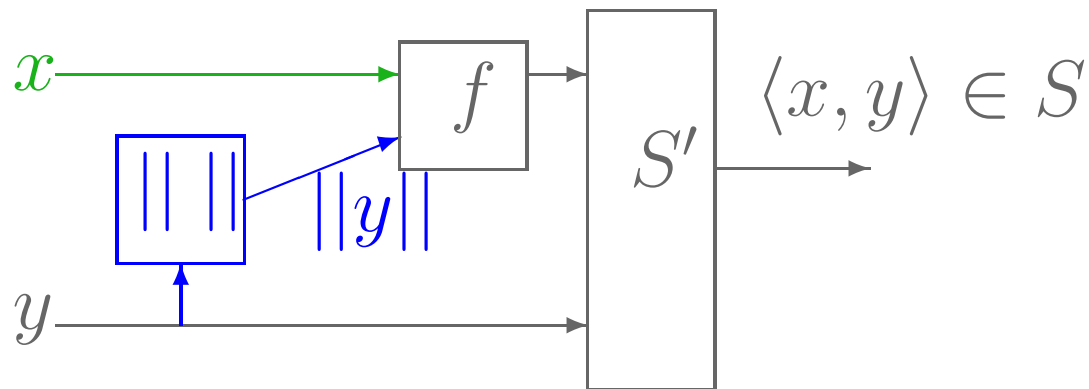


# Generalizing Techniques

# NU-Compilability

[Cadoli *et al.*, 1996a]  $\Rightarrow$  [2002]

*Non-Uniform* compilability

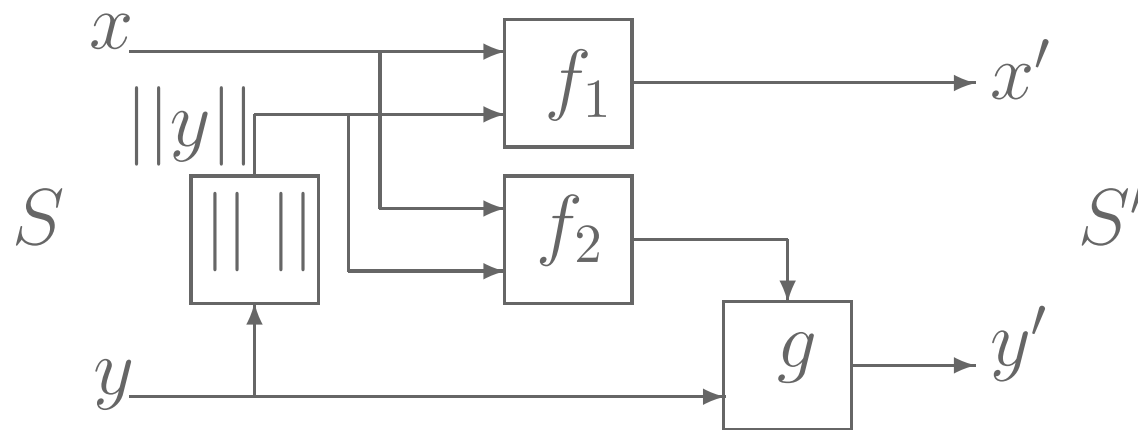


Combines:

- the  $\mathcal{P}/\text{poly}$  machine (size of  $y$ )
- the compilability machine (fixed part  $x$ )
- compilability implies nu-compilability

# Reductions

- Suppose problem  $S = \{\langle x, y \rangle\}$  can be reduced to problem  $S' = \{\langle x', y' \rangle\}$
- if  $S$  is non-compilable, so is  $S'$

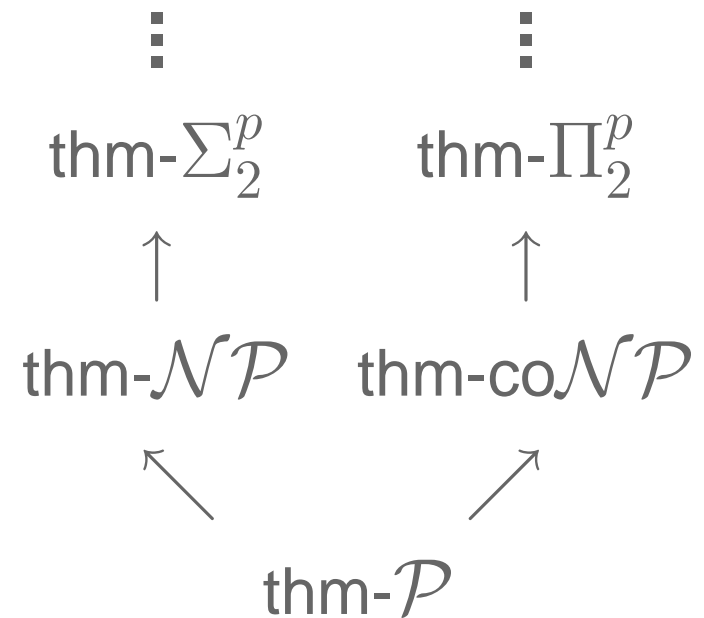
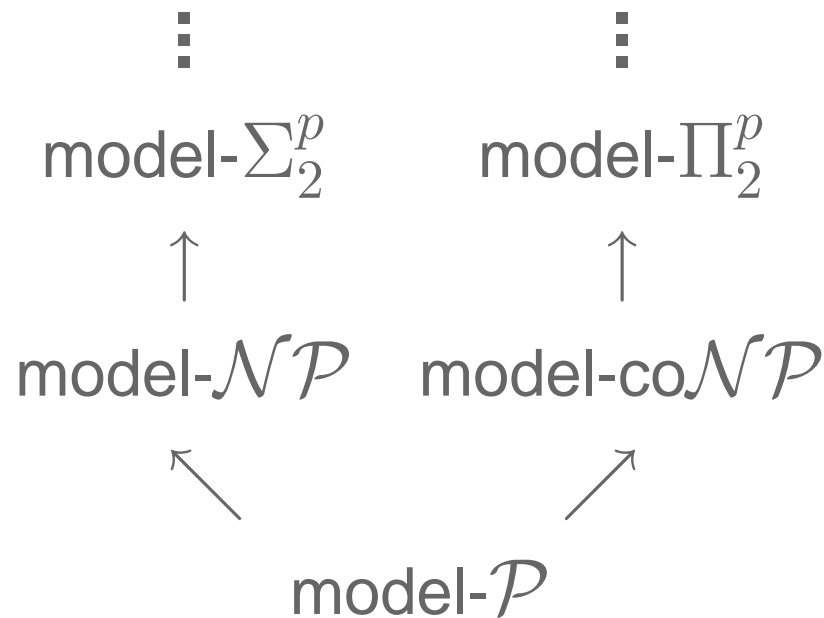


# Compilability Classes

Model checking

Theorem proving

compilable into...



# Space Efficiency of KR formalisms

	Time Complexity		<i>Space Efficiency</i>	
	Models	Theorems	Models	Theorems
Propositional Logic	$\mathcal{P}$	$\text{co}\mathcal{NP} \bullet$	model- $\mathcal{P}$	thm- $\text{co}\mathcal{NP} \bullet$
WIDTIO Belief Rev.	$\Sigma_2^p \bullet$	$\Pi_2^p \bullet$	model- $\mathcal{P}$	thm- $\text{co}\mathcal{NP} \bullet$
Skeptical Belief Rev.	$\text{co}\mathcal{NP} \bullet$	$\Pi_2^p \bullet$	model- $\text{co}\mathcal{NP} \bullet$	thm- $\Pi_2^p \bullet$
Circumscription	$\text{co}\mathcal{NP} \bullet$	$\Pi_2^p \bullet$	model- $\text{co}\mathcal{NP} \bullet$	thm- $\Pi_2^p \bullet$
GCWA	$\Delta_2^p[\log n] \downarrow$ $\text{co}\mathcal{NP} \uparrow$	$\Pi_2^p \bullet$	model- $\mathcal{P}$	thm- $\text{co}\mathcal{NP} \bullet$
Skeptical Default Reas.	$\Sigma_2^p \bullet$	$\Pi_2^p \bullet$	model- $\Sigma_2^p \bullet$	thm- $\Pi_2^p \bullet$
Credulous Default Reas.	N/A	$\Sigma_2^p \bullet$	N/A	thm- $\Sigma_2^p \bullet$
Stable Models in LP	$\mathcal{P}$	$\text{co}\mathcal{NP} \bullet$	model- $\mathcal{P}$	thm- $\text{co}\mathcal{NP} \bullet$

• — complete

↑ — hard

↓ — contained

# **Early 00's: Compilation goes on...**

# Database Theory for Compilation

[Cadoli and Mancini, 2002]

- express the data as relations
- express the problem as a second-order logic formula  $\varphi$
- find syntactic forms of  $\varphi$  such that the problem is compilable

# Compiling Propositional *KB* s

[Darwiche and Marquis, 2002]

- compile *KB* into NNF
- use NNF representation to answer queries
- best JAIR paper award (2006)



# Planning

- [Bäckström, 1995] relates planning formalism considering also the *size* of the shortest translation
- [Nebel, 2000] analyses *compilability* between planning formalisms
- [Liberatore, 2001] PLANSAT+ is not compilable, using general methods

# Abduction

[Liberatore and Schaerf, 2007]

- given  $KB$ , manifestations  $M$ , possible hypotheses  $H$
- find a subset  $H'$  of  $H$  such that
- $H' \cup KB \models M$
- $KB$  is fixed, so it makes sense to preprocess it
- yet all main problems (existence, relevance, necessity...) are not compilable

# Circuits

[Cadoli *et al.*, 2006]

- error-correcting circuits
- related to Hamming codes
- the size of  $k$ -error-correcting circuits cannot be a polynomial in  $k$
- related to Dalal's Belief Revision

**Goodbye, Marco...**

# References

In the notes of this slide,  
references can be found.

Slides are available at  
<http://sisinflab.poliba.it/donini>

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