# Abduction and Contraction in Description Logics

## *What, How, and Why*

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# Abstract

In the notes of this slide

The graft of Semantic Annotation into Electronic Commerce (EC) brings new opportunities for the application of Knowledge Representation techniques, originally devised for Knowledge Bases. Description Logics (DL) are one of the formal basis for Semantic Annotation, and the reasoning services they provide can be extended to cope with problems stemming from EC.

In this talk, I first give the EC scenario, recall the available Semantic Annotation technology, and highlight reasoning problems. Then, I introduce Concept Abduction and Concept Contraction as extensions of DL reasoning services.

In the second part of the talk, I present a Tableaux-based method to compute (some) abductions and contractions.

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- What next?

# Motivation

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(*P2P EC* from now on)

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- Marketplace: mostly, Web Site with human interaction

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- Marketplace: mostly, Web Site with human interaction
- Renowned example: eBay `http://www.ebay.com`

# Some figures

Did you ever tried to find . . .

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. . . how did you choose?

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. . . *which reasoning* did you employed?

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- the seller publishes offers
- the client browses...

- P2P: *Peer*-to-*Peer*
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- *both* parties can publish on the Web Site
- *Both* parties may take initiative (and browse...)

# Available Technology

# Semantic Annotation

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  — OWL - Web Ontology Language Overview
- *DAML* - DARPA Agent Markup Language
- Web Services can be described through languages like *DAML-S, OWL-S*,. . .

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  - formula consistent with $\mathcal{T} \cup \{O, R\}$

# Matchmaking

# What’s Matchmaking?

*First phase* in a Bilateral Commercial Transaction:

1. *Matchmaking* (find counterpart)
2. Negotiation (agree/tradeoff details)
3. Exchange (goods, services, money)

# Reasoning for Matchmaking

Which kind of reasoning is necessary for matching offers and demands?

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Do they match?

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*How well* they match? (compared to other offers/requests)

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—But: which reasoning?

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- *Posit* missing information
- *Revise* conflicting issues

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[Trastour *et al.*, 2002],[Di Noia *et al.*, 2003]
An offer $O$ and a request $R$ match...

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# Evaluating the match

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conflicting info: *430* vs. *360* (different models)

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in $R$, not in $O$: *Coupe/Spider, urgently required*

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in $O$, not in $R$: color *Argento, Bordeaux Leather* seats, *22,700* miles, $\ldots$

# Abduction and Contraction

# Abduction (history)

- C. S. Peirce (1839–1914)
  From $A \Rightarrow B$ and $B$, *abduce* $A$
- Abduction was the first step of scientific reasoning, the other two being
  - Deduction
  - Induction
- since [Pople, 1973] has been used to formalize Diagnosis in AI

# Abduction for Diagnosis— A crash course

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- symptoms $S$, *e.g.*, yellow-eyes
- a theory $T$, *e.g.*, hepatitis $\Rightarrow$ yellow-eyes, jaundice $\Rightarrow$ yellow-eyes

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- $T \wedge H \models S$

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Lots of minimality criteria!

[Eiter and Gottlob, 1995]

# Why Abduction for P2P EC— Intuition

| Request(buyer) | Offers (sellers) |
|---|---|
| | $O_1 = \boxed{\text{cellphone}}$ |
| $R = \boxed{\begin{array}{l}\text{cellphone,}\\ \text{bluetooth,}\\ \text{shutdown}\end{array}}$ | $O_2 = \boxed{\begin{array}{l}\text{cellphone,}\\ \text{bluetooth}\end{array}}$ |
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# Propositional Abduction 4 P2P EC

- Let $\mathcal{L}$ be a propositional language
- $\mathcal{H}$ be a subset of $\mathcal{L}$ (possible hypotheses)
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- Abd. 4 EC includes Abd. 4 Diag. when $O \equiv \top$ (no prior facts)

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  - Preference queries [Kießling, 2002] do not.

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    - *e.g.*, number of hypotheses in best $H$
    - *e.g.*, probability/plausibility of $H$
- construct an *explanation* for match suggestions
    - *e.g.*, a facilitator that suggests
      "Offer 213 seems to be the best, but requests *color:blue* and *Credit Card Payment* are not yet assessed"

# Concept Abduction

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- $\mathcal{L}$ be a Description Logic
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- language-specific
  - *e.g.*, minimal conjunctions if $\sqcup, \neg \notin \mathcal{H}$

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- $H_2 = \mathit{bundleOff}$ has *minimum length*
- neither solution is in the other set

# Intermezzo 1

## — Do you need a coffee?

# Belief Revision (history)

- [Gärdenfors, 1988]:
  Revise Knowledge $\mathcal{K}$ with new info $A$ by:
  1. *contracting* $\mathcal{K}$ into $\mathcal{K}^{-}_{\neg A}$ such that $\mathcal{K}^{-}_{\neg A} \not\models \neg A$
  2. *adding* $A$ to $\mathcal{K}^{-}_{\neg A}$
- Intuition: contract the least

# Belief Revision— another crash course!

$$\mathcal{K} = \left\{ \begin{array}{l} \mathit{1stFloor} \wedge \mathit{noSteps} \Rightarrow \mathit{easyAccess} \\ \mathit{1stFloor} \wedge \mathit{noSteps} \end{array} \right\}$$

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A *syntax-independent* revision:

$$\mathcal{K}_3^- = \left\{ \begin{array}{l} \mathit{1stFloor} \land \mathit{noSteps} \Rightarrow \mathit{easyAccess} \\ \mathit{1stFloor} \end{array} \right\}$$

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- *plus. . .*

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- . . . w.r.t. raw numbers

# Simple complexity results

When abducing $H?$ such that $C \sqcap H \sqsubseteq_{\mathcal{T}} D$,

$$H = \top \text{ iff } C \sqsubseteq_{\mathcal{T}} D \text{ already}$$

Note: $H = \top$ is both subsumption-maximal and minimum-length

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Min-Length Concept Abduction (every DL) is NP-hard for Definite Horn-Krom $\mathcal{T}$

[Colucci *et al.*, 2004]

# Simple complexity results (3)

When contracting $C \equiv_{\mathcal{T}} G \sqcap K$ such that $D \sqcap K$ is sat. w.r.t. $\mathcal{T}$,

$G = \top$ iff $C \sqcap D$ is already sat. w.r.t. $\mathcal{T}$

Note: $G = \top$ is both subsumption-maximal and minimum-length

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- drawback: how to *elicit* preferences & numbers?

# Alternatives to Abduction

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- drawback: cannot cope with inconsistencies (no Contraction)

# Outline of the talk

- ✓ Motivation: Electronic Commerce
- ✓ Abduction and Contraction: Definitions
  - ✓ Logical and computational properties
- A Tableaux-based calculus
- Implementation
- What next?

# Prefixed Tableaux

## ∼ for ∼

## Concept Abduction and Contraction

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  - concept formulas: $\begin{cases} \textbf{T}) \; x : C \\ \textbf{F}) \; x : C \end{cases}$
  - role formulas: $\begin{cases} \textbf{T})(x, xRn) : R \\ \textbf{F})(x, xRn) : \neg R \end{cases}$

# Assumptions

- $\mathcal{L}$ = $\mathcal{AL}$ (can be extended to $\mathcal{ALN}$)
- concepts are in Normal Form [Borgida and Patel-Schneider, 1994] (NF)
- $\mathcal{T}$ is normalized:
  $A \sqsubseteq B \sqcap C \quad \rightarrow \quad A \sqsubseteq B, A \sqsubseteq C$

# Tableaux Rules: $\sqcap$, $\sqcup$

$$\frac{\textbf{T})\ x : C \sqcap D}{\begin{array}{c}\textbf{T})\ x : C\\ \textbf{T})\ x : D\end{array}}\ \textbf{T}\sqcap \qquad\qquad \frac{\textbf{F})\ x : C \sqcup D}{\begin{array}{c}\textbf{F})\ x : C\\ \textbf{F})\ x : D\end{array}}\ \textbf{F}\sqcup$$

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(2 branches) (2 branches)

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$$\frac{\mathbf{T})\ x : \forall R.C \qquad \mathbf{T})(x, xRn) : R}{\mathbf{T})\ xRn : C}\ \mathbf{T}\forall_1$$

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Note: the label of the *concept formula* carries over!

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where $xRn$ is *new* in the branch / in the tableau

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forward-chain form$^*$

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backward-chain form$^*$

$$\frac{\begin{array}{c}\mathbf{F})\ x : C \\ A \sqsubseteq C \in \mathcal{T}\end{array}}{\mathbf{F})\ x : A}\ \mathbf{F} \sqsubseteq_2$$

# Tableaux Rules: inclusions

$$\frac{A \sqsubseteq C \in \mathcal{T}}{\textbf{F})\ x : A \sqcap NF(\neg C)}\ \textbf{F}\sqsubseteq$$

forward-chain form$^*$

$$\frac{\begin{array}{c}\textbf{F})\ x : \neg A \\ A \sqsubseteq C \in \mathcal{T}\end{array}}{\textbf{F})\ x : NF(\neg C)}\ \textbf{F}\sqsubseteq_1$$

backward-chain form$^*$

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(*) incomplete—useful for optimizations only

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Rules converting **T**) into **F**) and vice versa [Smullyan, 1968]

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are *not* present!

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(we need to trace a formula back to either $O$ or $R$)

# Clashes in a branch

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- a branch is closed if it contains a clash
- a tableau is closed if every branch is closed

# Start for Abduction

Find $H?$ such that $C \sqcap H \quad \sqsubseteq_{\mathcal{T}} \quad D$

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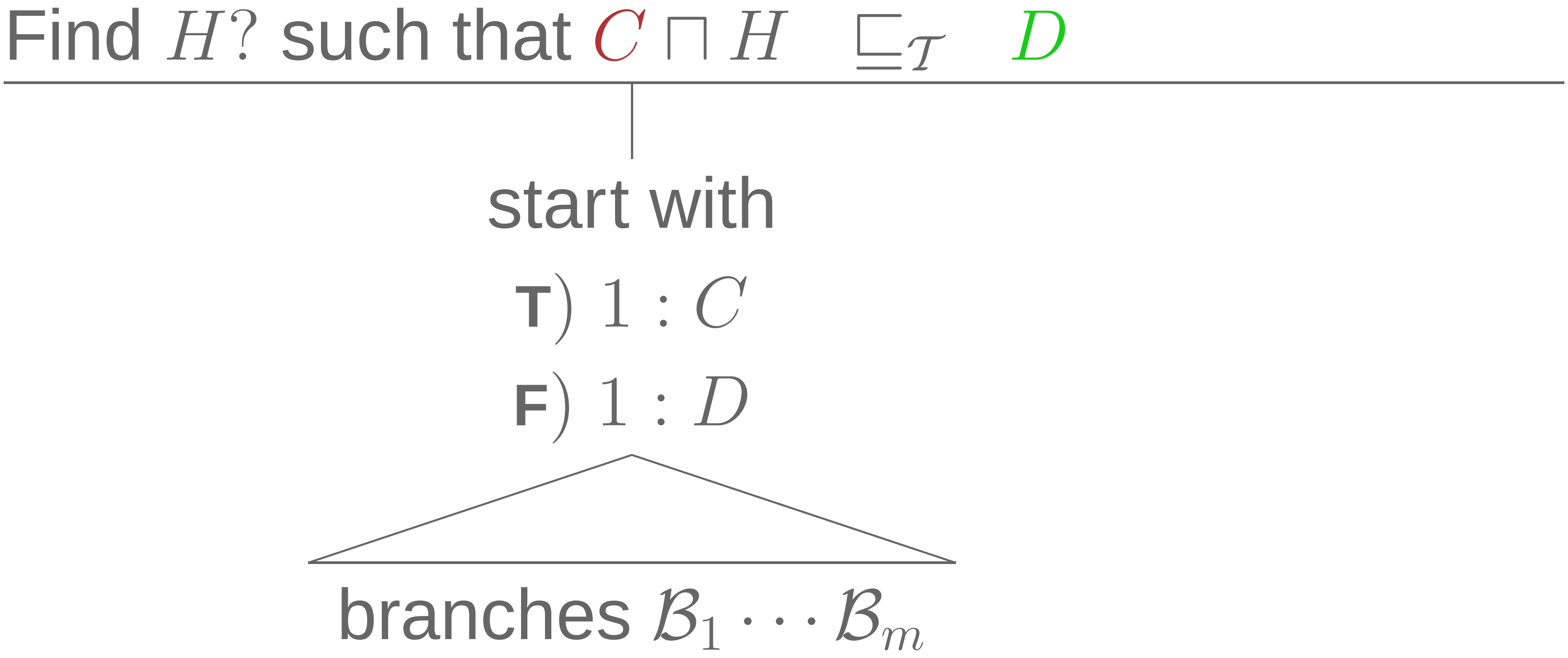

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start with

**T**) $1 : C$

**F**) $1 : D$

branches $\mathcal{B}_1 \cdots \mathcal{B}_m$

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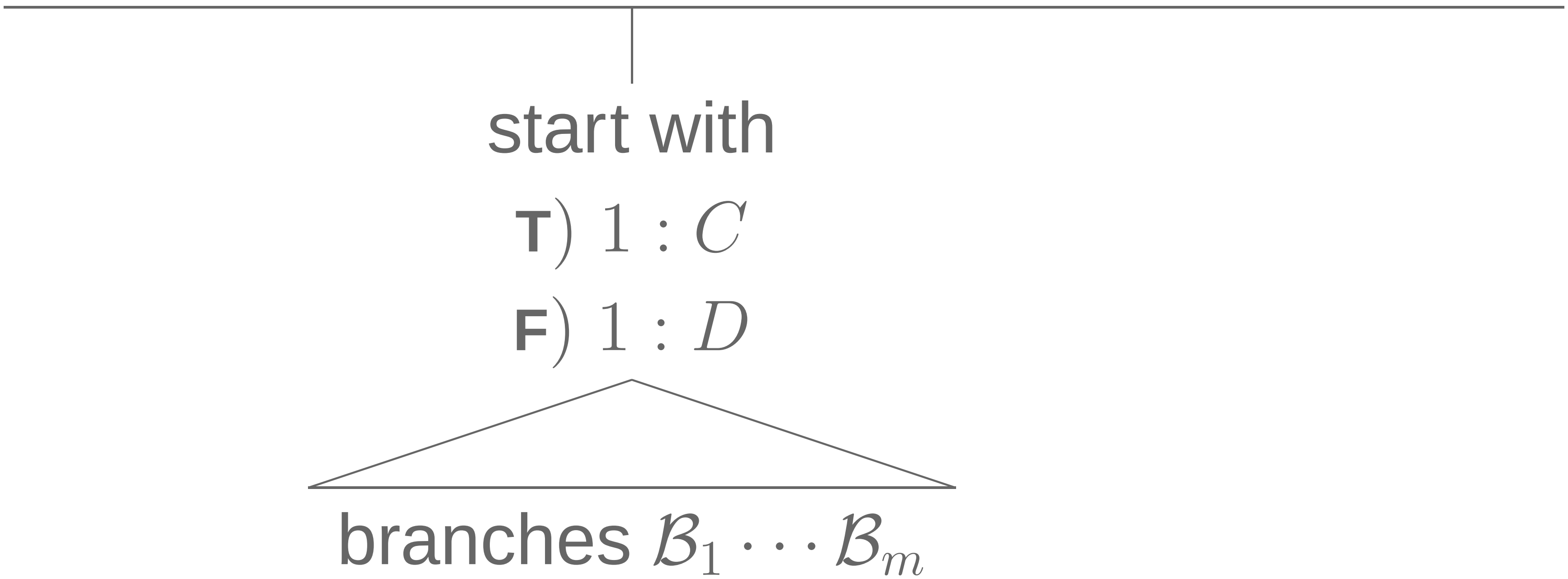

start with

**T**$)\ 1 : C$

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branches $\mathcal{B}_1 \cdots \mathcal{B}_m$

for every completed, open branch $\mathcal{B}$, *add* all formulas $\boxed{\mathbf{T})\ x : A}$ *ab.* or $\boxed{\mathbf{T})\ x : \neg A}$ *ab.* that yield a *heterogeneous* clash

# Computing Abduction

- *choose* one abducible $\boxed{\textbf{T})\ x_i : E_i}$ *ab.* for each open branch $\mathcal{B}_i$, for $i = 1, \ldots, m$

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Find $H?$ such that $A \sqcap B \sqcap H \quad \sqsubseteq_{\mathcal{T}} \quad B \sqcap C$

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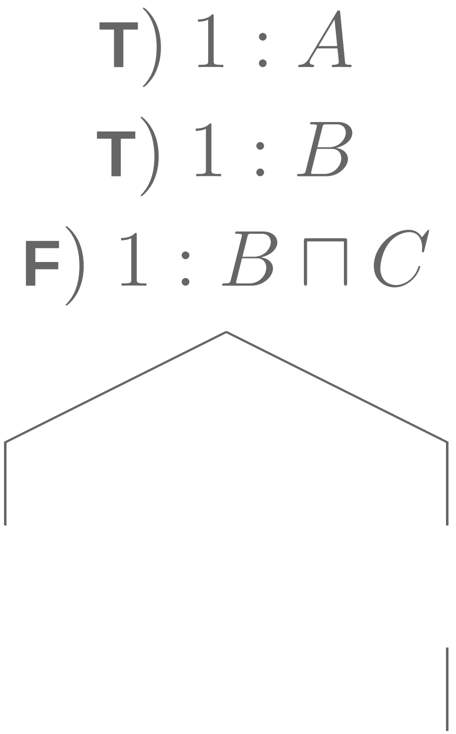

$\textbf{T}) \; 1 : A$

$\textbf{T}) \; 1 : B$

$\textbf{F}) \; 1 : B \sqcap C$

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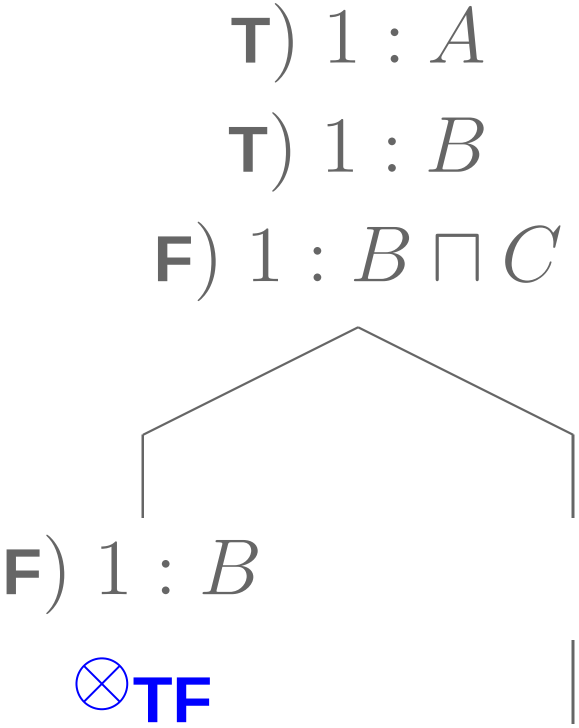

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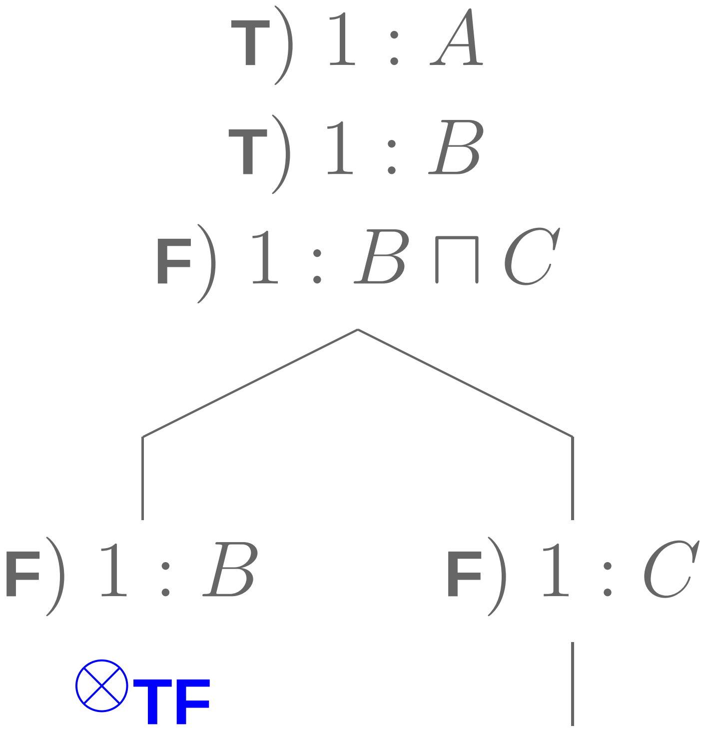

**T**) $1 : A$

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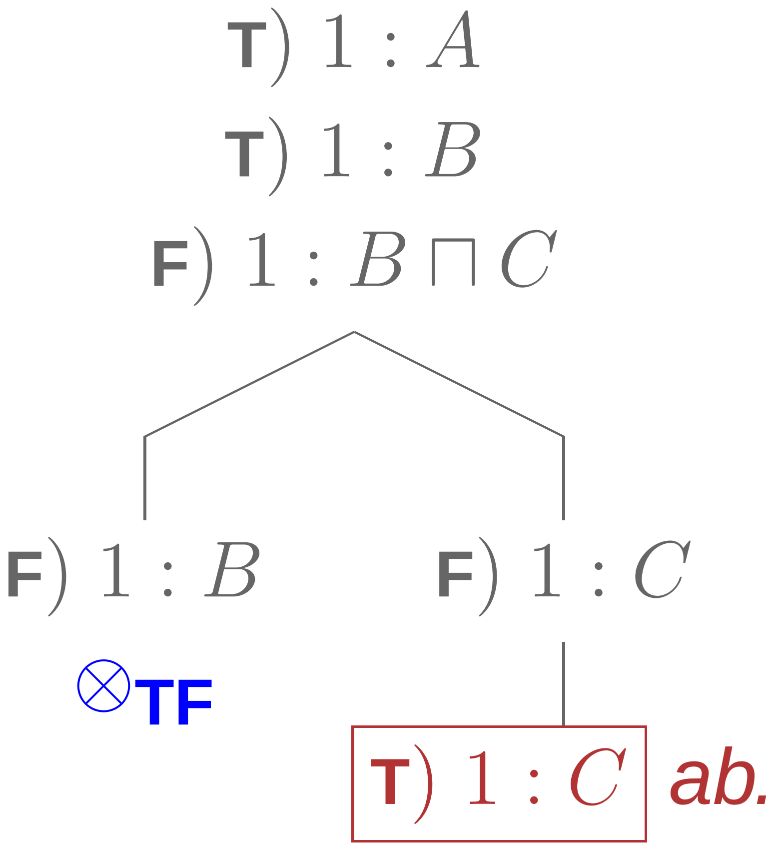

**T**) $1 : A$

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**F**) $1 : B$ $\otimes$**TF**

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$\boxed{\textbf{T}) \; 1 : C}$ *ab.*

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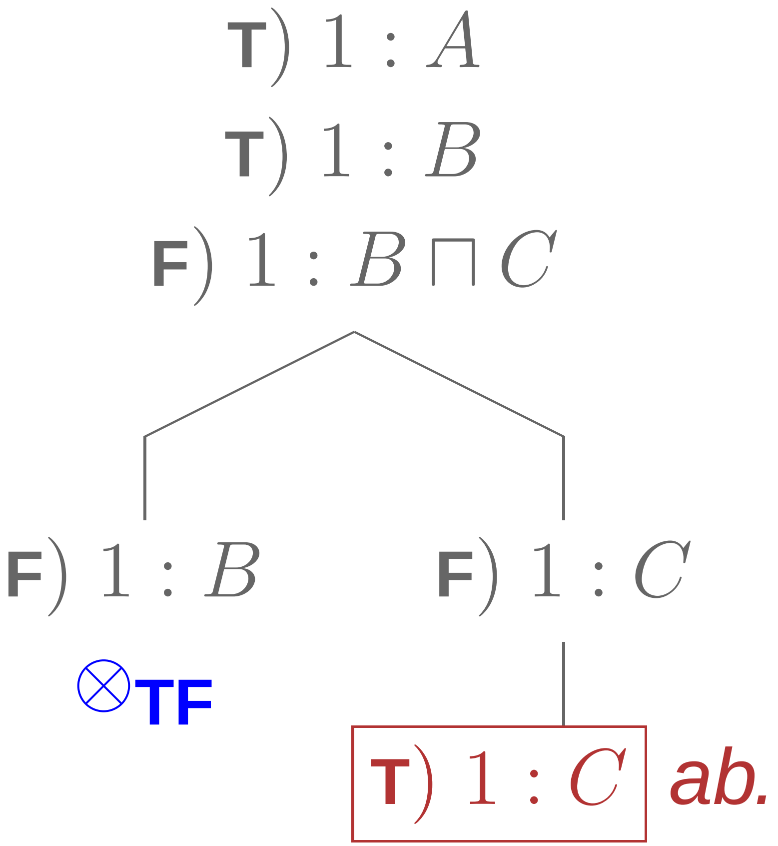

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**F**) $1 : B$

$\otimes$**TF**

**F**) $1 : C$

$\boxed{\textbf{T}) \; 1 : C}$ *ab.*

One solution

$H =$

$= \forall roles(1).C$

$= \forall \epsilon.C$

$= C$

# Two open branches, one choice

Find $H$? such that $A \sqcap \forall R.B \sqcap H \quad \sqsubseteq_{\mathcal{T}} \quad C \sqcap \forall R.D$

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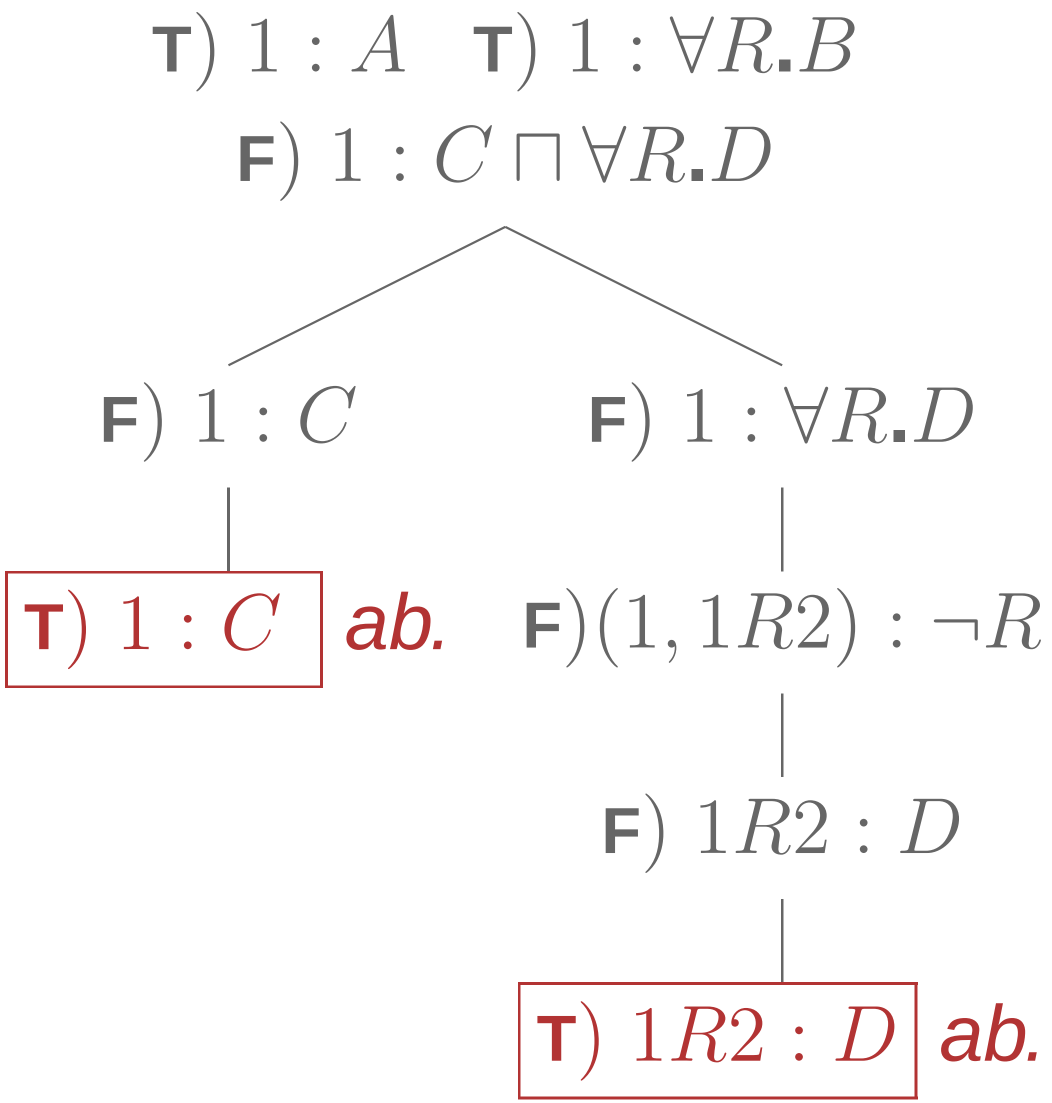

**T**) $1 : A$ **T**) $1 : \forall R.B$

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$\boxed{\mathbf{T})\ 1 : C}$ *ab.*

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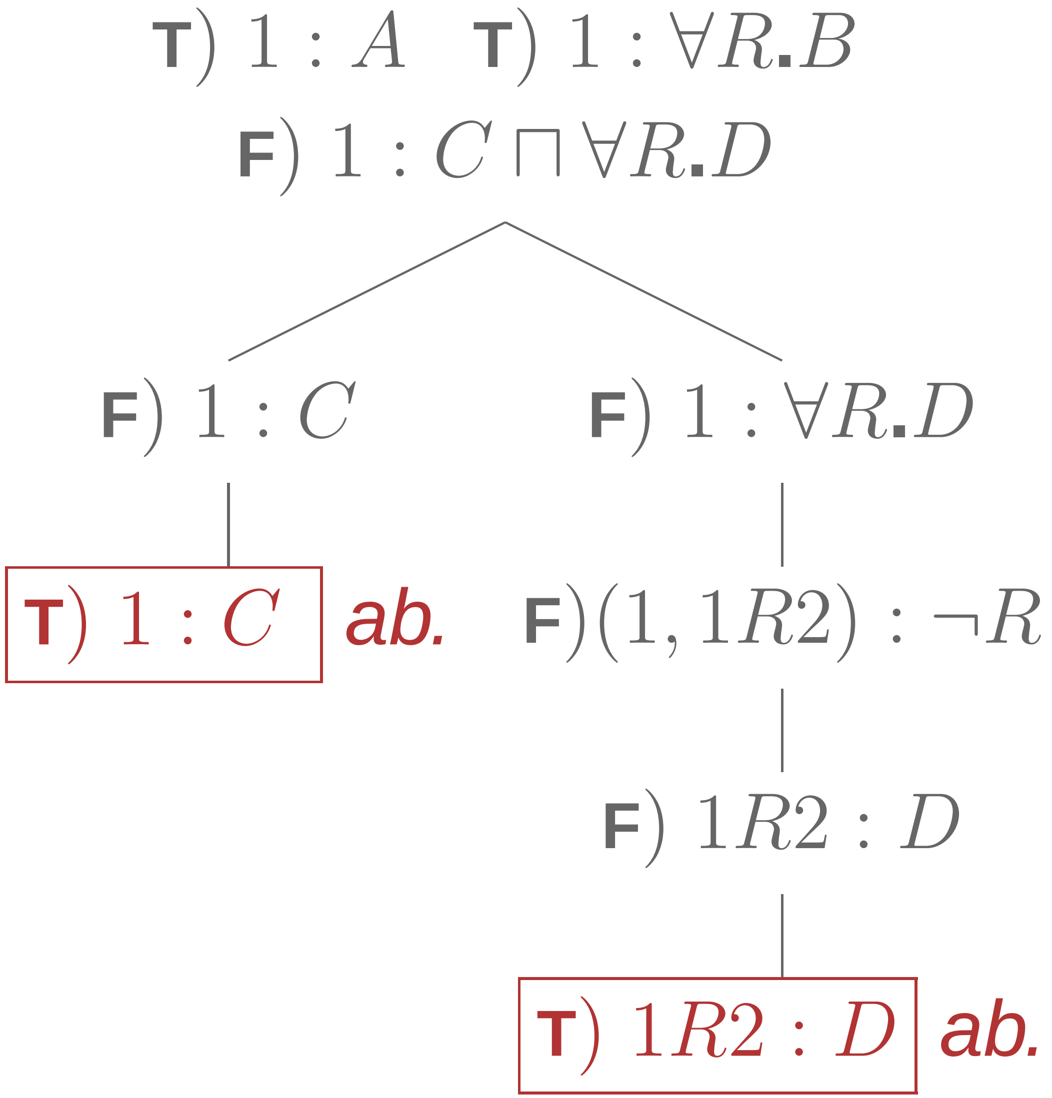

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**T**) $1 : C$ *ab.*

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**T**) $1R2 : D$ *ab.*

One solution

$H = \forall roles(1).C \sqcap \forall roles(1R2).D = C \sqcap \forall R.D$

# One open branch, two choices

Find $H?$ such that $A \sqcap H \quad \sqsubseteq_{\mathcal{T}} \quad B$ with $\mathcal{T} = \{D \sqsubseteq B\}$

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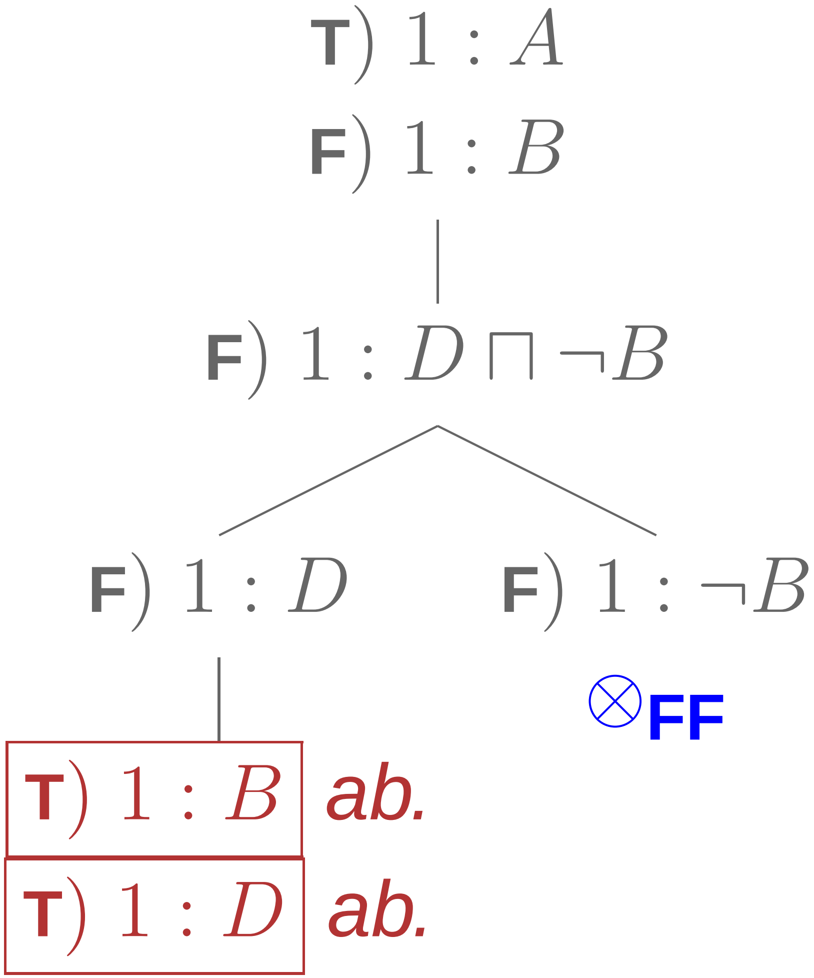

**T**$)\ 1 : A$

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**F**$)\ 1 : D \sqcap \neg B$

**F**$)\ 1 : D$

**F**$)\ 1 : \neg B$

$\otimes$**FF**

**T**$)\ 1 : B$ *ab.*

**T**$)\ 1 : D$ *ab.*

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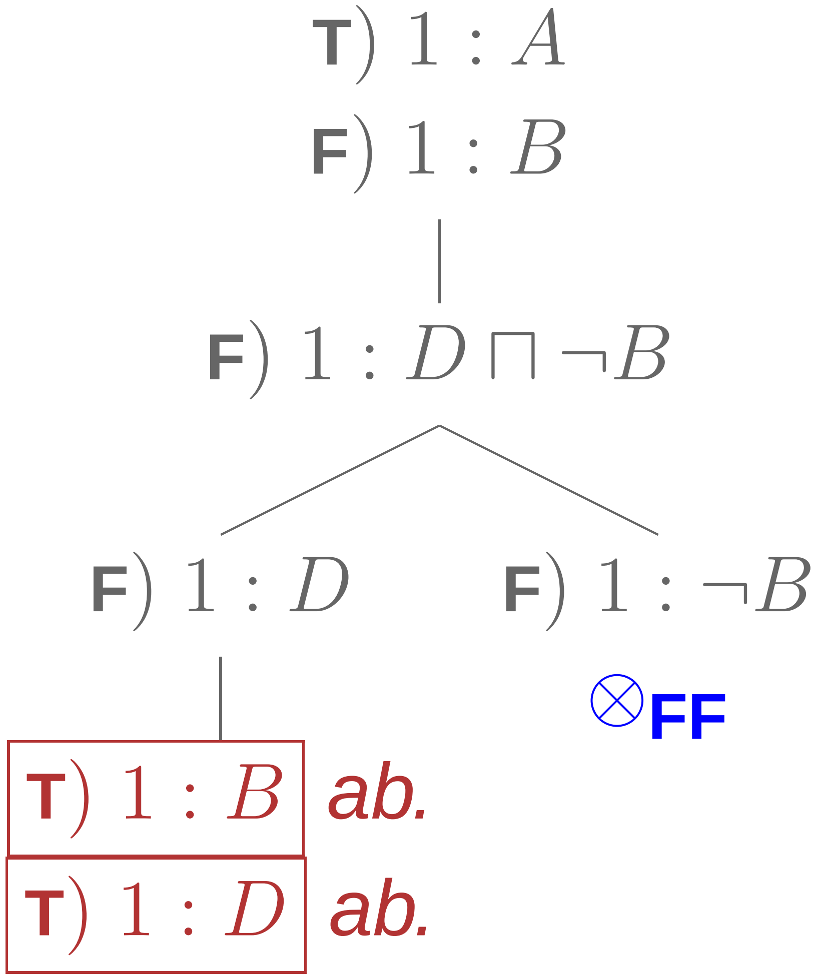

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**F**) $1 : D$ **F**) $1 : \neg B$

$\otimes$**FF**

$\boxed{\textbf{T}) \ 1 : B}$ *ab.*

$\boxed{\textbf{T}) \ 1 : D}$ *ab.*

Two solutions
$H_1 = B$,
$H_2 = D$

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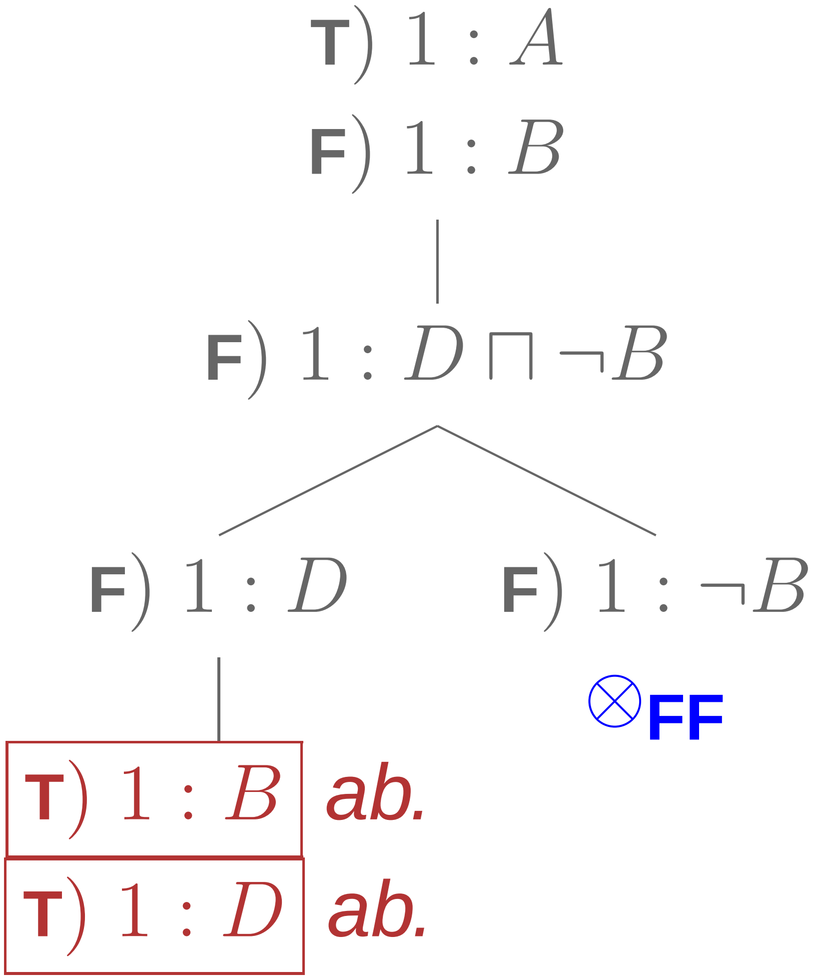

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**F**) $1 : D$ **F**) $1 : \neg B$

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$\boxed{\textbf{T}) \; 1 : B}$ *ab.*

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Two solutions
$H_1 = B$,
$H_2 = D$

Note: $H_2$ is *not* a solution of $B - LCS(A, B)$

[Benatallah *et al.*, 2005],

[Lécué and Delteil, 2007]

# 2 open branches, 2 choices each

Find $H?$ s.t. $(\mathit{FiatPanda}\sqcap)\mathit{yr2000} \sqsubseteq_{\mathcal{T}}$

$(\mathit{FiatPanda}\sqcap)\mathit{radio} \sqcap \mathit{fogLmps}$

with $\mathcal{T} = \{\mathit{bundleOff} \sqsubseteq \mathit{radio} \sqcap \mathit{fogLmps}\}$

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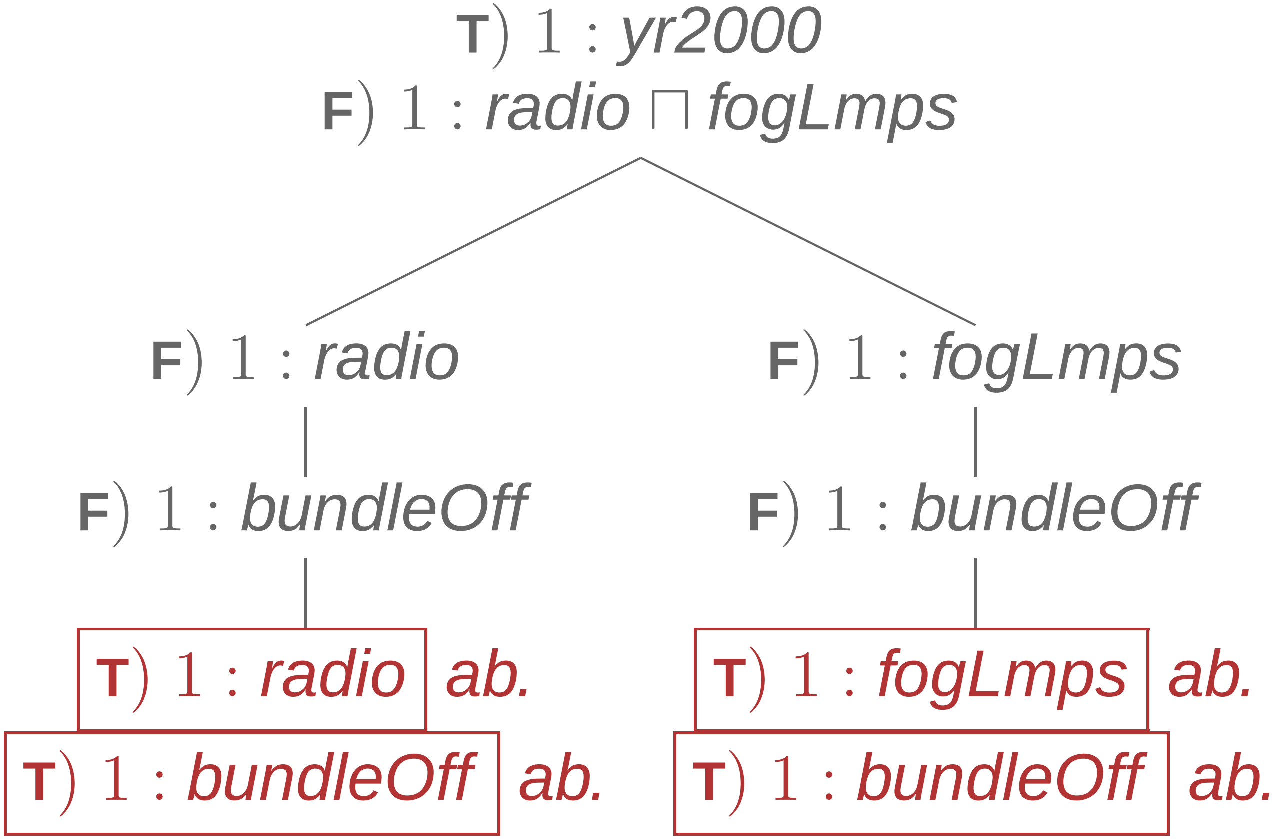

**T**$)\ 1 : \textit{yr2000}$

**F**$)\ 1 : \textit{radio} \sqcap \textit{fogLmps}$

**F**$)\ 1 : \textit{radio}$ | **F**$)\ 1 : \textit{fogLmps}$

**F**$)\ 1 : \textit{bundleOff}$ | **F**$)\ 1 : \textit{bundleOff}$

$\boxed{\textbf{T})\ 1 : \textit{radio}}$ *ab.* | $\boxed{\textbf{T})\ 1 : \textit{fogLmps}}$ *ab.*

$\boxed{\textbf{T})\ 1 : \textit{bundleOff}}$ *ab.* | $\boxed{\textbf{T})\ 1 : \textit{bundleOff}}$ *ab.*

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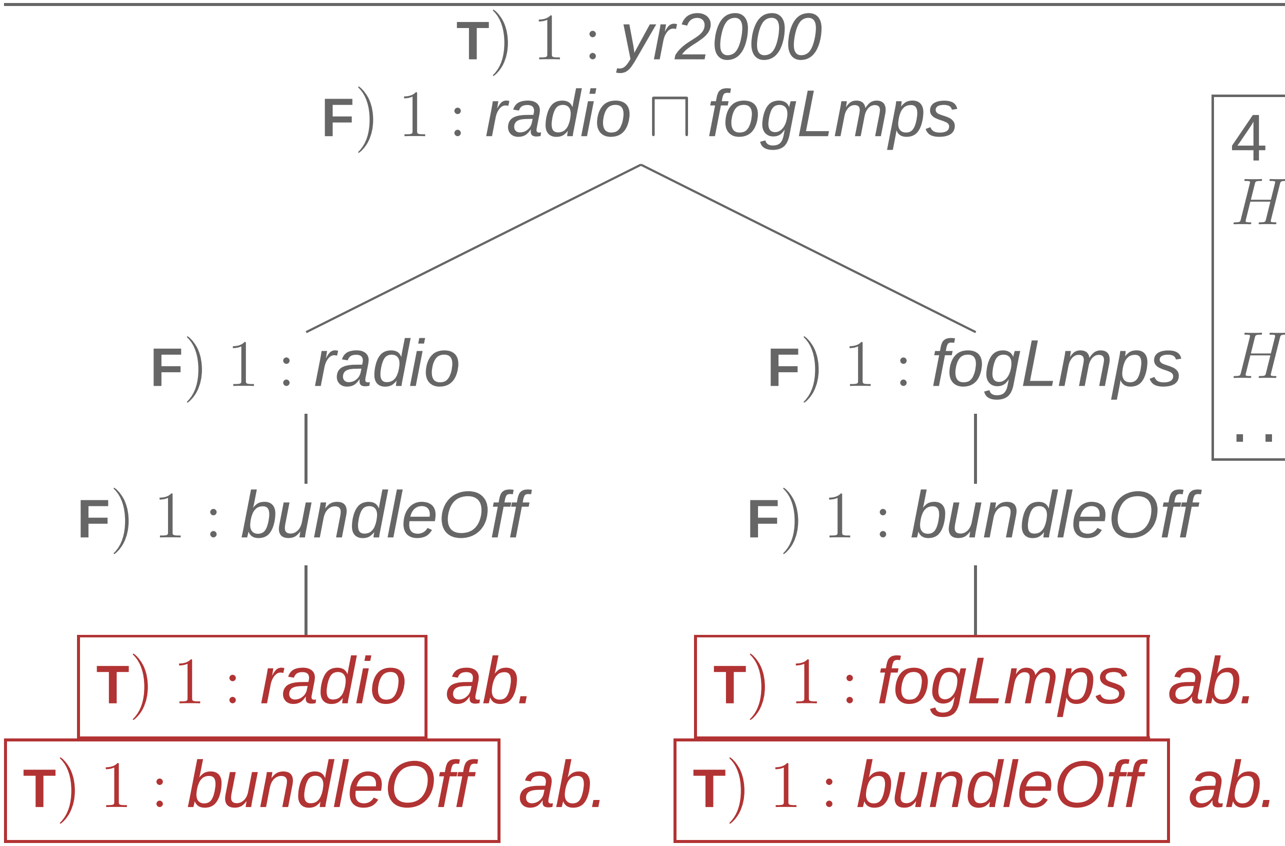

**T**$)\ 1 :$ *yr2000*

**F**$)\ 1 :$ *radio* $\sqcap$ *fogLmps*

**F**$)\ 1 :$ *radio*

**F**$)\ 1 :$ *fogLmps*

**F**$)\ 1 :$ *bundleOff*

**F**$)\ 1 :$ *bundleOff*

**T**$)\ 1 :$ *radio* *ab.*

**T**$)\ 1 :$ *fogLmps* *ab.*

**T**$)\ 1 :$ *bundleOff* *ab.*

**T**$)\ 1 :$ *bundleOff* *ab.*

4 solutions:

$H_1 =$ *radio*$\sqcap$ *fogLmps*

$H_2 =$ *bundleOff*

$\ldots$

# Properties

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- subsumption-maximal $H$
    - after choosing $H$, keep applying rules
    - $H$ is *not* subs-max iff every branch closes *also* with another clash

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branches $\mathcal{B}_1 \cdots \mathcal{B}_m$,

complete

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**T**) $1 : C$

**F**) $1 : NF(\neg D)$

branches $\mathcal{B}_1 \cdots \mathcal{B}_m$, complete

*choose* one $\mathcal{B}$ with *heterogeneous* clashes only

# Computing Contraction

- take all $\langle E_i, x_i \rangle$ s.t. $\boxed{\textbf{T}) \; x_i : E_i}$ $\boxed{\textbf{F}) \; x_i : NF(\neg E_i)}$ is a heterogeneous clash in the chosen $\mathcal{B}$

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- several $\langle G, K\rangle$'s, depending on the chosen $\mathcal{B}$
- Note: $\exists R$ is contracted only if it clashes with $\forall R.\bot$

# Two branches

Find $G, K?$ s.t. $\neg$*satAlarm* $\sqcap K$ is sat. and *alarm* $\sqcap$ *GPS* $\equiv_{\mathcal{T}}$ $G \sqcap K$, with $\mathcal{T} = \{$*alarm* $\sqcap$ *GPS* $\sqsubseteq$ *satAlarm*$\}$

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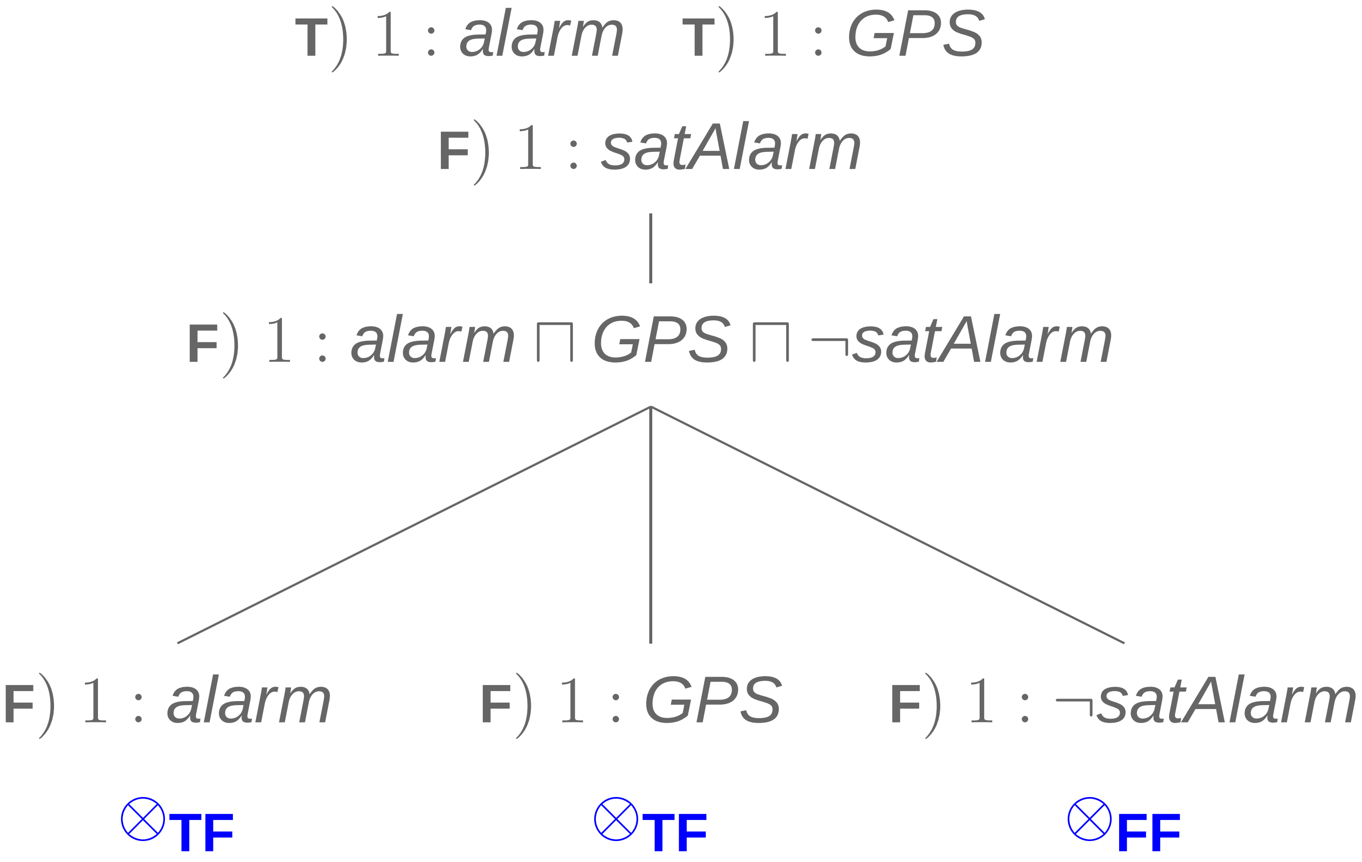


**T**) $1 :$ *alarm*   **T**) $1 :$ *GPS*

**F**) $1 :$ *satAlarm*

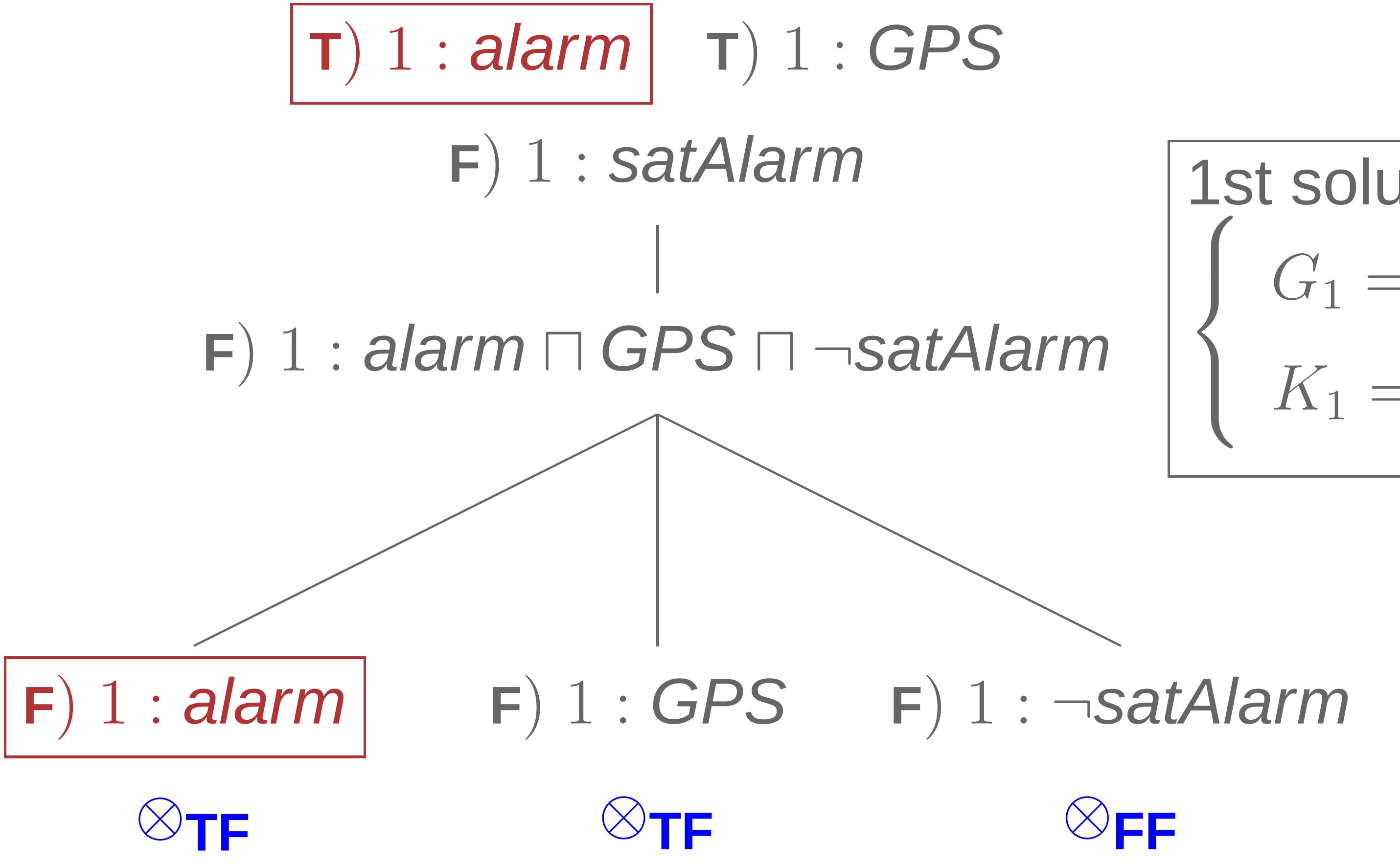
**F**) $1 :$ *alarm* $\sqcap$ *GPS* $\sqcap \neg$*satAlarm*

**F**) $1 :$ *alarm*   **F**) $1 :$ *GPS*   **F**) $1 : \neg$*satAlarm*

$\otimes$**TF**   $\otimes$**TF**   $\otimes$**FF**

# Two branches

Find $G, K?$ s.t. $\neg$*satAlarm* $\sqcap K$ is sat. and *alarm* $\sqcap$ *GPS* $\equiv_{\mathcal{T}} G \sqcap K$, with $\mathcal{T} = \{$*alarm* $\sqcap$ *GPS* $\sqsubseteq$ *satAlarm*$\}$



**T**) $1 :$ *alarm* **T**) $1 :$ *GPS*

**F**) $1 :$ *satAlarm*

**F**) $1 :$ *alarm* $\sqcap$ *GPS* $\sqcap \neg$*satAlarm*

**F**) $1 :$ *alarm* **F**) $1 :$ *GPS* **F**) $1 : \neg$*satAlarm*

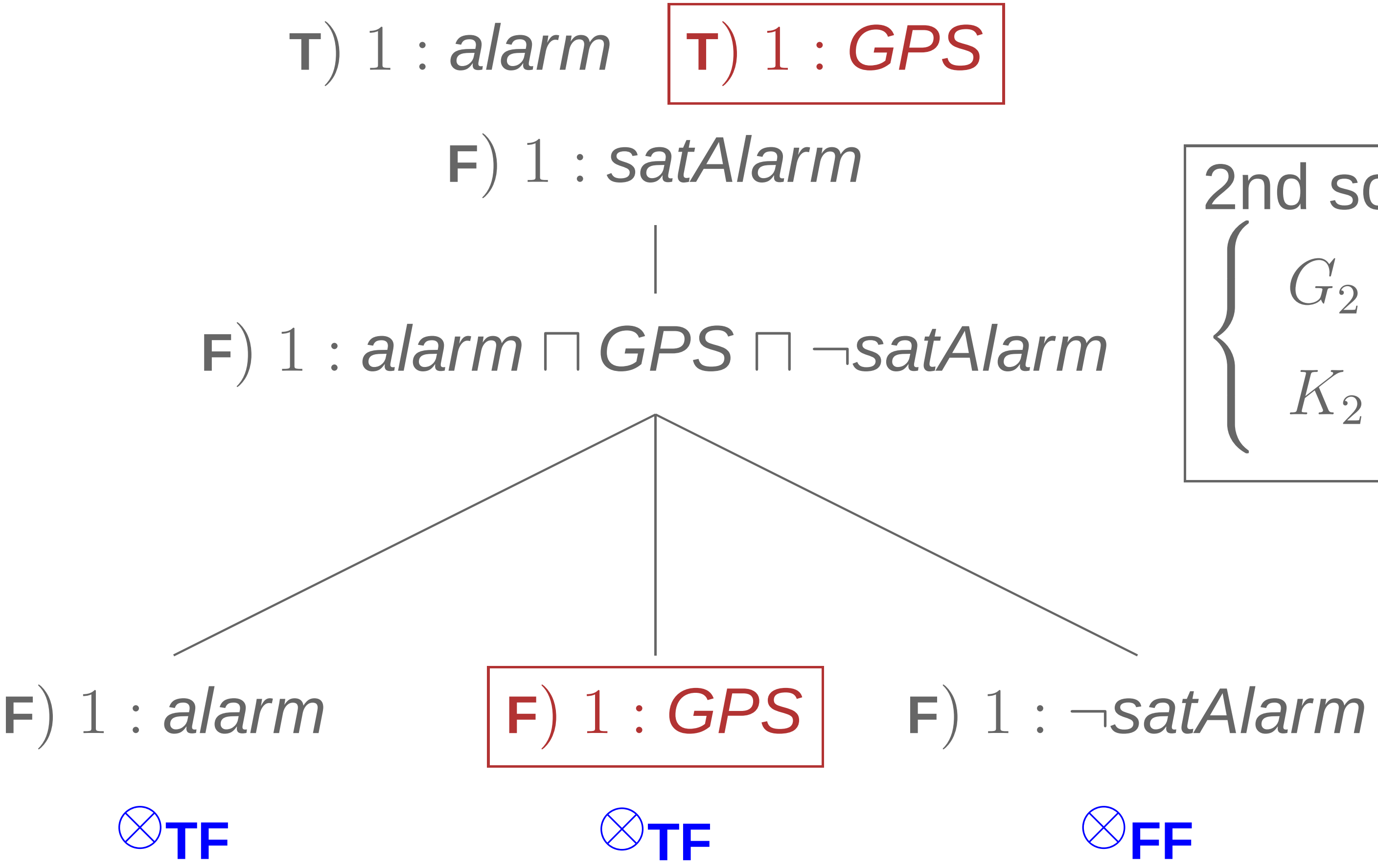
$\otimes$**TF** $\otimes$**TF** $\otimes$**FF**

1st solution:

$$\begin{cases} G_1 = \textit{alarm} \\ K_1 = \textit{GPS} \end{cases}$$

# Two branches

Find $G, K?$ s.t. $\neg \mathit{satAlarm} \sqcap K$ is sat. and $\mathit{alarm} \sqcap \mathit{GPS} \equiv_{\mathcal{T}} G \sqcap K$, with $\mathcal{T} = \{\mathit{alarm} \sqcap \mathit{GPS} \sqsubseteq \mathit{satAlarm}\}$



**T**) $1 : \mathit{alarm}$ $\boxed{\textbf{T}) \ 1 : \mathit{GPS}}$

**F**) $1 : \mathit{satAlarm}$

**F**) $1 : \mathit{alarm} \sqcap \mathit{GPS} \sqcap \neg \mathit{satAlarm}$

**F**) $1 : \mathit{alarm}$ $\boxed{\textbf{F}) \ 1 : \mathit{GPS}}$ **F**) $1 : \neg \mathit{satAlarm}$

$\otimes$**TF** $\otimes$**TF** $\otimes$**FF**

2nd solution:

$$\begin{cases} G_2 = \mathit{GPS} \\ K_2 = \mathit{alarm} \end{cases}$$

# Intermezzo 2

**— Now, *I* need a coffee...**

# Implementation

# MaMaS-tng

- MAtch MAking Service - The Next Generation
- Subsumption, Satisfiability, Concept Abduction and Concept Contraction in $\mathcal{ALN}$
- exposes an exended DIG 1.1 interface
- available as an HTTP service (only HTTP-POST requests)

# OwlEd

- OWL Editor
- supports MaMaS-tng
- also other reasoners endowed of DIG1.1 interface
- OwlEd beta is freely downloadable

# Next issues

- more expressive DLs ($\mathcal{ALC}$)

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- more expressive DLs ($\mathcal{ALC}$)
- fuzzy DLs for concrete domains [Ragone *et al.*, 2007], [Ragone *et al.*, 2008]
  - *e.g.*, price, color, delivery time
  - the mediator can *negotiate* conflicting issues

# Future issues

- agents carry *both* an offer *and* a request

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  - “Award-winner chinese calligrapher seeks flat in London” — Sunday Times, August 2002
  - Dating services
- epistemic statements
  - “Best prices paid”
  - “smokers allowed”

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- Azzurra Ragone, Michele Ruta,
- Floriano Scioscia, Eufemia Tinelli,
- ... among many others

# References

In the notes of this slide,
references can be found.

Slides are available at
http://sisinflab.poliba.it/donini

# References

[Benatallah *et al.*, 2005] Boualem Benatallah, Mohand-Said Hacid, Alain Léger, Christophe Rey, and Farouk Toumani. On automating web services discovery. *VLDB J.*, 14(1):84–96, 2005.

[Borgida and Patel-Schneider, 1994] A. Borgida and P. F. Patel-Schneider. A Semantics and Complete Algorithm for Subsumption in the CLASSIC Description Logic. *Journal of Artificial Intelligence Research*, 1:277–308, 1994.

[Calì *et al.*, 2004] A. Calì, D. Calvanese, S. Colucci, T. Di Noia, and F. M. Donini. A description logic based approach for matching user profiles. In *Proceedings of the 17th International Workshop on Description Logics (DL'04)*, volume 104 of *CEUR Workshop Proceedings*, 2004.

[Cialdea Mayer and Pirri, 1995] Marta Cialdea Mayer and Fiora Pirri. Propositional abduction in modal logic. *Logic Journal of the IGPL*, 3(6):907–919, 1995.

[Colucci *et al.*, 2004] S. Colucci, T. Di Noia, E. Di Sciascio, F.M. Donini, and M. Mongiello. Uniform tableaux-based approach to concept abduction and contraction in aln dl. In *Proceedings of the 17th International Workshop on Description Logics (DL'04)*, volume 104 of *CEUR Workshop Proceedings*, 2004.

[Di Noia *et al.*, 2003] T. Di Noia, E. Di Sciascio, F.M. Donini, and M. Mongiello. A system for principled Matchmaking in an electronic marketplace. In *Proc. International World Wide Web Conference (WWW '03)*, pages 321–330. ACM Press, 2003.

[Di Noia *et al.*, 2007] Tommaso Di Noia, Eugenio Di Sciascio, and Francesco M. Donini. Semantic matchmaking as non-monotonic reasoning: A description logic approach. *Journal of Artificial Intelligence Research*, 29(2), 2007.

[Donini and Massacci, 2000] F.M. Donini and F. Massacci. Exptime tableaux for $\mathcal{ALC}$. *Artificial Intelligence*, 124:87–138, 2000.

[Eiter and Gottlob, 1995] T. Eiter and G. Gottlob. The complexity of logic-based abduction. *Journal of the ACM*, 42(1):3–42, 1995.

[Gärdenfors, 1988] P. Gärdenfors. *Knowledge in Flux: Modeling the Dynamics of Epistemic States*. Bradford Books, MIT Press, Cambridge, MA, 1988.

[Kießling, 2002] W. Kießling. Foundations of preferences in database systems. In *Proc. of 28th VLDB Conference*, 2002.

[Lécué and Delteil, 2007] Freddy Lécué and Alexandre Delteil. Making the difference in semantic web service composition. In *AAAI*, pages 1383–1388. AAAI Press, 2007.

[Levesque, 1989] H. J. Levesque. A knowledge-level account of abduction. In *Proceedings of the Eleventh International Joint Conference on Artificial Intelligence (IJCAI'89)*, pages 1061–1067, 1989.

[Lukasiewicz and Schellhase, 2006] Thomas Lukasiewicz and Jörg Schellhase. Variable-strength conditional preferences for matchmaking in description logics. In Patrick Doherty, John Mylopoulos, and Christopher A. Welty, editors, *KR*, pages 164–174. AAAI Press, 2006.

[Pople, 1973] Harry E. Pople. On the mechanization of abductive logic. In *IJCAI*, pages 147–152, 1973.

[Ragone *et al.*, 2007] Azzurra Ragone, Umberto Straccia, Tommaso Di Noia, Eugenio Di Sciascio, and Francesco M. Donini. Vague knowledge bases for matchmaking in p2p e-marketplaces. In Enrico Franconi, Michael Kifer, and Wolfgang May, editors, *ESWC*, volume 4519 of *Lecture Notes in Computer Science*, pages 414–428. Springer, 2007.

[Ragone *et al.*, 2008] Azzurra Ragone, Umberto Straccia, Tommaso Di Noia, Eugenio Di Sciascio, and Francesco M. Donini. Towards a fuzzy logic for automated multi-issue negotiation. In Sven Hartmann and Gabriele Kern-Isberner, editors, *FoIKS*, volume 4932 of *Lecture Notes in Computer Science*, pages 381–396. Springer, 2008.

[Reggia *et al.*, 1985] James A. Reggia, Dana S. Nau, Pearl Y. Wang, and Yun Peng. A formal model of diagnostic inference, ii. algorithmic solution and application. *Information Sciences*, 37(1-3):257–285, 1985.

[Smullyan, 1968] R. M. Smullyan. *First Order Logic*. Springer-Verlag, Berlin, 1968.

[Teege, 1994] G. Teege. Making the difference: A subtraction operation for description logics. In *Proceedings of the Fourth International Conference on the Principles of Knowledge Representation and Reasoning (KR'94)*, pages 540–550. MK, 1994.

[Trastour *et al.*, 2002] D. Trastour, C. Bartolini, and C. Priest. Semantic web support for the Business-to-Business e-commerce lifecycle. In *Proc. International World Wide Web Conference (WWW) '02*, pages 89–98. ACM, 2002.

[Zaremski and Wing, 1997] Amy Moormann Zaremski and Jeannette M. Wing. Specification matching of software components. *ACM Trans. Softw. Eng. Methodol.*, 6(4):333–369, 1997.